Cartel Detection and Collusion Screening: An Empirical Analysis of the London Metal Exchange *

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Abstract

In order to fight collusive behaviors, the best scenario for competition authorities would be the possibility to analyze detailed information on firms' costs and prices, being the price-cost margin a robust indicator of market power. However, information on firms' costs is rarely available. In this context, a fascinating technique to detect data manipulation and rigged prices is offered by an odd phenomenon called Benford's Law, otherwise known as First-Digit Law, which has been successfully employed to discover the "Libor Scandal" much time before the opening of the cartel settlement procedure. Thus, the main objective of the present paper is to apply a such useful instrument to track the price of the aluminium traded on the London Metal Exchange, following the allegations according to which there would be an aluminium cartel behind. As a result, quick tests such as Benford's Law can only be helpful to inspect markets where price patterns show signs of collusion. Given the budget constraints to which antitrust watchdogs are commonly subject to, a such price screen could be set up, just exploiting the data available, as warning system to identify cases that require further investigations.

Keywords: Benford's Law, Cartel Detection, Collusion Screening, Competition Authorities, Data Manipulation, Monopolization, Oligopolistic Markets, Price Fixing, Variance Screen.

JEL Classification: C10; D40; L13; L41

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≪In Francia abbiamo seguito le vostre elezioni.
Il capo del governo ha tre reti televisive?≫

≪Sì≫.

≪Perché in Francia non si potrebbe, c'è una legge.
Voi non avete la legge antitrust?≫

≪Sì. Sì e no. Più no che sì≫.

Nanni Moretti

1 Libor Scandal

In 2013, the European Commission imposed an administrative fine of 1.7 billion euro to some of the world's largest banking companies involved in what has been described by the mass media as "Libor Scandal"[1]. The record sanction, being the highest ever levied by the officials of Brussels for a cartel infringement, was issued to 8 international financial institutions for participating in illegal agreements relating to interest rate derivatives. As it is common knowledge, interest rate derivatives are financial products, such as futures, options, swaps, which are both employed as insurance tools for managing the risk of interest rate fluctuations and traded worldwide as investment assets by financial intermediaries. The value of these financial derivatives comes from the level of a benchmark interest rate, such as the Euro Interbank Offered Rate (Euribor), which is used for the euro area, or the London Interbank Offered Rate (Libor), which is used for several currencies including the Japanese Yen. In turn, the value of these benchmarks reflects the averaged interest rate at which, respectively, a selected panel of Eurozone and London banks offer to lend funds in a given currency to other banks on the daily interbank market.

In a nutshell, the cartel aimed at manipulating the pricing process of the Euribor and the Libor, distorting the competition in the underlying trading of interest rate derivatives. Since at least $800 trillion in derivatives, loans, securities and other financial products are tied to the Euribor and the Libor, such was the dimension of the scandal, which *inter alia* has highlighted the urgency of a regulatory reform of the banking sector, the largest one to have been rigged so far.

[1] Commission Decision of 4 December 2013, *Euro Interest Rate Derivatives*, Case AT.39914; Commission Decision of 4 December 2013, *Yen Interest Rate Derivatives*, Case AT.39861; European Commission, *Antitrust: Commission fines banks € 1.71 billion for participating in cartels in the interest rate derivatives industry*, Press Release, IP/13/1208, 4 December 2013, Brussels, Belgium.

2 Benford's Law

A crucial expedient for revealing the "Libor Scandal" has been the leniency program, joined by a member of the cartel at issue providing an active cooperation in the investigation of the Commission in exchange of full immunity. Beyond the success of the cartel settlement procedure and the relevant dimension of the market involved, from a competition policy standpoint, the Libor case offers another interesting food for thought, being an excellent example of how antitrust authorities can employ screening instruments to identify collusive behaviors.

A fascinating technique to detect rigged prices is offered by an odd phenomenon called Benford's Law, otherwise known as First-Digit Law. Although a primordial statement must be attributed to Newcomb (1881)[2], in a 1938 paper, the father of the law, a physicist working at General Electric, recognized the existence of a specific pattern that often occurs in vast datasets[3]. In particular, the law consists in a frequency distribution which describes the probability according to which a number present in a random dataset starts with a certain digit.

Theoretically, if a set of numbers were truly random, each leading digit would appear about 11% of the time. On the contrary, Benford's Law predicts a logarithmic weakly monotonic distribution, according to which the digit 1 occurs as leading digit about 30% of the time, while larger digits occur in that position less and less frequently (cf. Formula 1). In other terms, the leading digits are not distributed evenly, as it would be natural to expect, but following a distribution where 1 is the most frequent and 9 is the less common. The law, which has also been generalized to digits beyond the first, tends to be more precise in datasets which exhibit multiple orders of magnitude and for types of values which grow exponentially.

Formula 1 - *Benford's Law Logarithmic Probability Distribution Function*

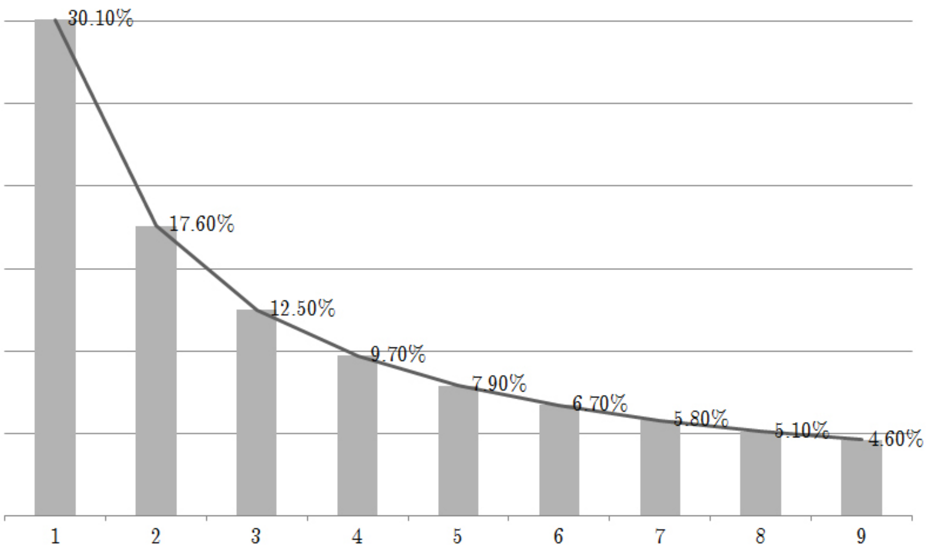
$$P(d) = log_{10}(d+1) \text{ - } log_{10}(d) = log_{10}\ (\tfrac{d+1}{d}) = log_{10}\ (1+\tfrac{1}{d})$$

[2]Newcomb, S. (1881), *Note on the Frequency of Use of the Different Digits in Natural Numbers*, American Journal of Mathematics, Vol. 4, No. 1, The Johns Hopkins University Press, Baltimore, United States, pp. 39-40.

[3]Benford, F. (1938), *The Law of Anomalous Numbers*, Proceedings of the American Philosophical Society, Vol. 78, No. 4, American Philosophical Society, Philadelphia, United States, pp. 551-572.

A brief and intuitive explanation of why the law naturally occurs is that usually we start counting from the digit 1 until the digit 9. It is obvious that if we think to the digits from 1 to 9, we have the same probability that a random number starts with any of these digits. But if we consider a range of numbers, for example from 1 to 20, we count more numbers starting with the digit 1. The same happens if we consider the range of numbers from 1 to 30, where we count many numbers starting with the digit 1, but also many others starting with the digit 2. In any case, what matters is that, in order to have many numbers starting with the digit 9, it is necessary to examine a large dataset. As a result, analyzing for instance distributions of numbers related to populations or surfaces, the probability to have a number starting with the digit 1 will be higher than that to have a number with 9 as leading digit. Accordingly, Benford showed that, for several types of distributions, the probability that a number starts with a certain digit tends to be always the same (cf. Figure 1).

Figure 1 - *Probability Distribution of Leading Digits according to Benford's Law*



3 Literature Review

The predictive power of Benford's Law has been ascertained valid in several situations normally observable in the real world. Death rates, election votes, financial transitions, government spendings, income distributions, physical and mathematical constants, population numbers and stock prices are just few examples over which the law applies. It is not a case that auditors have successfully employed it to detect frauds and manipulations in accounting data since the 1970s. At that time also Varian (1972)[4] suggested the possibility to use the law to uncover falsifications in socio-economic data collected for public purposes, under the assumption that who aims at rigging datasets tends to provide numbers distributed according to a uniform pattern. Nigrini (1999)[5] as well showed that the law can be exploited for taxation controls, after having tested it with success on real cases of fiscal scams.

Thus, given its regularity, the law can be adopted to test economic data in several cases. It's application is rather straightforward: even though a dataset has been artificially ordered in such a way to preserve randomness, the distribution of the digits will definitely violate the pattern predicted by the law. Within the present framework, in a seminal paper by Abrantes-Metz *et al.* (2011)[6], the authors considered worthwhile to test the theory over Libor data, using the second digit distribution variant of the law. The surprising result was that the benchmark interest rate at issue departed significantly from the Benford's Law pattern over an extended period of time, signaling the possibility of a rate manipulation. As a result, through a quick application of the test, the Libor cartel could have been discovered much time before the opening of the settlement procedure.

In Brähler *et al.* (2011)[7], a Benford's Law test was applied to investigate the quality of macroeconomic data reported by the EU member states to Eurostat in order to comply with the Stability and Growth Pact criteria. Since government statistics are comparable in nature to financial accounting, governments, like firms towards auditors, might be tempted to adjust the national account balances, given the strict obligations to which are subject to. The authors of the study found that the official statistics submitted by Greece registred the greatest deviation from the expected Benford's Law distribution in comparison to all the other EU countries.

[4]Varian, H.R. (1972), *Benford's Law (Letters to the Editor)*, The American Statistician, Vol. 26, Issue 3, Taylor & Francis Journals, London, United Kingdom, pp. 62-65.

[5]Nigrini, M.J. (1999), *I've Got Your Number: How a Mathematical Phenomenon Can Help CPAs Uncover Fraud and Other Irregularities*, Journal of Accountancy, Vol. 187, Issue 5, American Institute of Certified Public Accountants, New York, United States, pp. 15-27.

[6]Abrantes-Metz, R.M., Judge, G., Villas-Boas, S. (2011), *Tracking the Libor Rate*, Applied Economics Letters, Vol. 10, Issue 10, Taylor & Francis Journals, London, United Kingdom, pp. 893-899.

[7]Brähler, G., Engel, S., Göttsche, M., Rauch, B. (2011), *Fact and Fiction in EU-Governmental Economic Data*, German Economic Review, Vol. 12, Issue 3, John Wiley & Sons, New York, United States, pp. 243-255.

The manipulation of financial data by the Greek institutions has officially been certified by the Commission at a later stage.

4 Empirical Analysis of the London Metal Exchange

As well as for the "Libor Scandal", the Wall Street Journal has launched in 2011 an investigation about possible anti-competitive practices on the London Metal Exchange (LME), allegedly resulting in artificially high prices for a category of commodities, above all aluminium, whose hoarding is managed by a group of international banking corporations[8]. In the last years, in fact, the metals industry has been invested by a huge wave of acquisitions by major investment banks of metals warehouses located around the world. According to the allegations, the "too big to fail banks" at issue, being owners of large aluminium warehouses able to hoard massive supply in excess, despite the record levels of production registred since 2011, would have deliberately delayed the market distribution of aluminium products, inflating so its final price and gathering in the meantime exorbitant rents and other fees for the storage operations (cf. Figure 2). As a matter of fact, aluminium is the only commodity for which prompt delivery is not guaranteed, requiring instead its distribution several months. In the past, an analogous supply bottleneck strategy was used by De Beers to obstruct the market clearing and to maintain the price of diamonds extremely high.

The group of bank holding companies owners of the aluminium warehouses has replied to the allegations asserting that it merely follows the guidelines established by the LME (although, it is worth to notice, the investment banks in question are members and shareholders of the LME itself). However, in the United States, three class-action lawsuits were filed in 2013 against the alleged aluminium cartel for its presumed control over the LME. According to the plaintiffs, 5 billion dollars has been the total cost of the cartel at the expense of consumers for the period 2011-2013.[9]

[8]Hotter, A., Shumsky. T. (2011), *Wall Street Gets Eyed in Metal Squeeze. Some Say Warehousing Inflates Prices*, The Wall Street Journal, 17 June 2001, New York, United States.

[9]Master Screens Inc., *et al.* v. Goldman Sachs Group Inc., *et al.*, Case No. 13-CV-00431, United States District Court for the Northern District of Florida, Tallahassee Division; Superior Extrusion Inc., *et al.* v. Goldman Sachs Group Inc., *et al.*, Case No. 13-CV-13315, United States

Figure 2 - *LME Aluminium Price (US$ per Tonne) (2002-2013)*

As a result, the main objective of the present paper is to apply Benford's Law to track the daily LME aluminium price over the period 2011-2013, in order to verify possible data manipulations which would suggest the existence of a cartel. We analyze this time period for two basic reasons: firstly, because after the collapse of aluminum prices due to the global financial crisis, 2011 is the year when the commodity at issue began to reach very high prices, apparently not justified by the record levels of production touched; secondly, because the suspect of a possible collusion started to be felt at that time.

For what concerns the application of Benford's Law, since the nominal value of the aluminium price does not change very much over short periods of time, an analysis of the first digit distribution would be meaningless since the pattern predicted by the law would be definitely violated. On the contrary, in Table 2 and 3, as well as in Figure 3 and Figure 4, we can observe how the empirical second digit distribution and in particular the empirical third digit distribution of the LME aluminium price for the period 2002-2013 follow the expected pattern. This confirms the fact that the price spans the nine digit space only if we consider the second and the third digit.

District Court for the Eastern District of Michigan, Detroit Division; River Parish Contractors Inc., *et al.* v. Goldman Sachs Group Inc., *et al.*, Case No. 13-CV-05267, United States District Court for the Eastern District of Louisiana, New Orleans Division.

Table 1 - *Bendord's Law Second Digit Test: LME Aluminium (US$ per Tonne) (2002-2013)*

Digit	LME Frequency	Benford Rate	LME Rate	Δ
0	226	11.97%	7.48%	-4.49%
1	157	11.39%	5.20%	-6.19%
2	191	10.82%	6.32%	-4.50%
3	447	10.43%	14.80%	4.36%
4	445	10.03%	14.73%	4.70%
5	209	9.67%	6.92%	-2.75%
6	239	9.34%	7.91%	-1.43%
7	377	9.04%	12.48%	3.44%
8	451	8.76%	14.93%	6.17%
9	2	8.50%	9.24%	0.74%
Total	3,021			

Table 2 - *Bendord's Law Third Digit Test: LME Aluminium (US$ per Tonne) (2002-2013)*[10]

Digit	LME Frequency	Benford Rate	LME Rate	Δ
0	338	11.97%	11.19%	-0.78%
1	298	11.39%	9.86%	-1.52%
2	294	10.82%	9.73%	-1.09%
3	333	10.43%	11.02%	0.59%
4	313	10.03%	10.36%	0.33%
5	269	9.67%	8.90%	-0.76%
6	291	9.34%	9.63%	0.30%
7	312	9.04%	10.33%	1.29%
8	285	8.76%	9.43%	0.68%
9	288	8.50%	9.53%	1.03%
Total	3,021			

[10]In Table 2, Benfor's Law second digit rates, rather than third digit ones, are reported since, for the sake of simplicity of computations, we deleted first digits from the entire dataset when we dealt with the third digit distribution. Thus, we can treat third digits as they were second digits. Of course, this expedient does not alter the value of Benford's Law analysis. Furthermore, this allow us to better and equally compare the two LME Aluminium sets of rates, reported in Table 1 and Table 2, respect to Benford's Law one.

Figure 3 - *Bendord's Law Second Digit Test: LME Aluminium (US$ per Tonne) (2002-2013)*

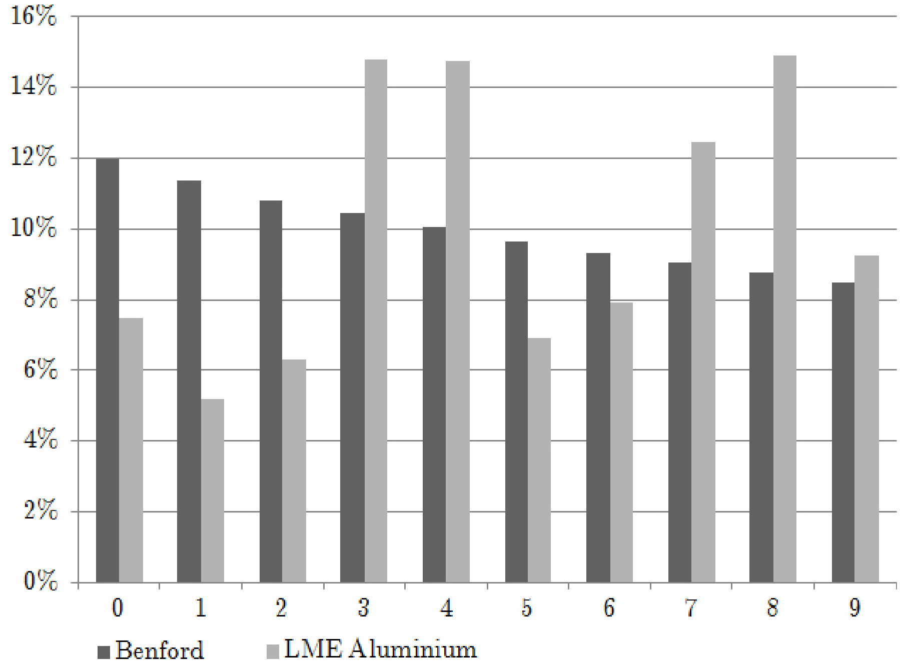
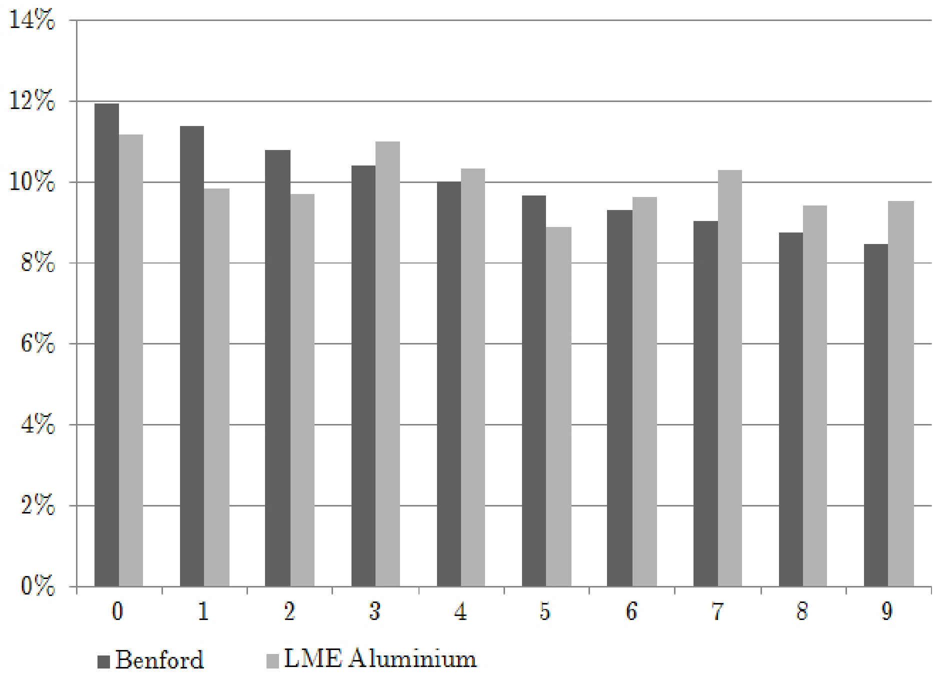


Figure 4 - *Bendord's Law Third Digit Test: LME Aluminium (US$ per Tonne) (2002-2013)*



Since our main concern is with the aluminium price trend registred in the last 3 years, following the methodology adopted to detect the "Libor Scandal", we now proceed to test the closeness to the Benford's Law pattern of the empirical distributions of both the second and third digits of the daily LME aluminium price. Our assessment is based on rolling six month periods, starting from 2011 until 2013. In particular, in Table 3, we present the empirical frequencies of the second digit distribution, whilst in Table 4 we present the empirical frequencies of the third digit distribution. The two tables at issue, given the size, are reported at the end of the current work.

Two are the main results. As for the second digit distribution, we register empirical frequencies which depart significantly from the expected Benford's Law pattern. This raises potential concerns relative to the data integrity of the aluminium price. Only the empirical frequencies of the third digit distribution, which we decided to compute for a more conservative analysis, are in line with the pattern predicted by the law. In this regard, it is important to remind that for the Libor cartel, it was enough to find departures similar to those we found in the empirical second digit distribution of the LME aluminium price to raise alert about possible rigged prices. Furthermore, in the Libor case, several periods of time where Benford's Law was respected were found in any case. Here, instead, at least for the period 2011-2013, the deviation from the expected pattern is costant. Moreover, the deviation of both the second and third digit distributions from the Benford's Law pattern tends to increase over time, reaching the maximum levels in 2013. The opening of an investigation to further examine the aluminium industry, according to the price screen at issue, seems therefore worthwhile.

5 Policy Conclusions

In order to detect and fight cartel conducts, the best scenario for competition agencies would be of course the possibility to analyze detailed information on firms' costs and prices, being the price-cost margin a robust indicator of market power. However, information on firms' costs is rarely available. In this context, as pointed out by our analysis of the London Metal Exchange, quick tests such as Benford's Law can only be helpful to inspect markets where price patterns show signs of collusion. Given the budget constraints to which antitrust watchdogs are commonly subject to, a such price screen could be set up, just exploiting the data available, as warning system to identify cases that require the opening of an investigation.

Nevertheless, it is important to underline that Benford's Law, as any other statistical test, cannot be adopted as investigative tool in any circumstance, since not all real data are expected to obey the law at issue. The risk otherwise is to fall into false-positive assessments. In any case, what seems certain is the fact that such a useful instrument, if not employed by competition authorities, will be surely used by firms to further disguise and mystify cartel activities.

Table 3 - *Bendord's Law and LME Aluminium Second Digit Frequencies over rolling 6-Month Periods from 2011 to 2013*

Digit	Benford Rate	01/2011 06/2011 (1)	Δ	02/2011 07/2011 (2)	Δ	03/2011 08/2011 (3)	Δ	04/2011 09/2011 (4)	Δ	05/2011 10/2011 (5)	Δ	06/2011 11/2011 (6)	Δ
0	11.97%	0.00%	-11.97%	0.00%	-11.97%	0.00%	-11.97%	0.00%	-11.97%	0.00%	-11.97%	6.92%	-5.04%
1	11.39%	0.00%	-11.39%	0.00%	-11.39%	0.00%	-11.39%	3.20%	-8.19%	16.41%	5.02%	23.08%	11.69%
2	10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%	3.20%	-7.62%	6.25%	-4.57%	6.15%	-4.67%
3	10.43%	4.88%	-5.55%	0.00%	-10.43%	11.11%	0.68%	20.80%	10.37%	20.31%	9.88%	20.00%	9.57%
4	10.03%	27.64%	17.61%	25.00%	14.97%	22.22%	12.19%	20.80%	10.77%	20.31%	10.28%	18.46%	8.43%
5	9.67%	37.40%	27.73%	42.74%	33.07%	34.13%	24.46%	21.60%	11.93%	19.53%	9.86%	13.08%	3.41%
6	9.34%	24.39%	15.05%	26.61%	17.28%	26.98%	17.65%	24.80%	15.46%	15.63%	6.29%	9.23%	-0.11%
7	9.04%	5.69%	-3.34%	5.65%	-3.39%	5.56%	-3.48%	5.60%	-3.44%	1.56%	-7.47%	0.00%	-9.04%
8	8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%
9	8.50%	0.00%	-8.50%	0.00%	-8.50%	0.00%	-8.50%	0.00%	-8.50%	0.00%	-8.50%	3.08%	-5.42%

Digit	Benford Rate	07/2011 12/2011 (7)	Δ	08/2011 01/2012 (8)	Δ	09/2011 02/2012 (9)	Δ	10/2011 03/2012 (10)	Δ	11/2011 04/2012 (11)	Δ	12/2011 05/2012 (12)	Δ
0	11.97%	10.94%	-1.03%	14.84%	2.88%	14.96%	2.99%	15.75%	3.78%	15.75%	3.78%	30.40%	18.43%
1	11.39%	27.34%	15.95%	36.72%	25.33%	44.09%	32.71%	51.97%	40.58%	51.97%	40.58%	32.00%	20.61%
2	10.82%	6.25%	-4.57%	9.38%	-1.45%	17.32%	6.50%	19.69%	8.86%	19.69%	8.86%	16.80%	5.98%
3	10.43%	20.31%	9.88%	20.31%	9.88%	11.02%	0.59%	1.57%	-8.86%	1.57%	-8.86%	1.60%	-8.83%
4	10.03%	14.06%	4.03%	5.47%	-4.56%	1.57%	-8.46%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%
5	9.67%	7.03%	-2.64%	1.56%	-8.11%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%
6	9.34%	3.13%	-6.21%	0.78%	-8.56%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%
7	9.04%	0.00%	-9.04%	0.00%	-9.04%	0.00%	-9.04%	0.00%	-9.04%	0.00%	-9.04%	0.00%	-9.04%
8	8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%	0.00%	-8.76%
9	8.50%	10.94%	2.44%	10.94%	2.44%	11.02%	2.52%	11.02%	2.52%	11.02%	2.52%	19.20%	10.70%

Digit	Benford Rate	01/2012 06/2012 (13)	Δ	02/2012 07/2012 (14)	Δ	03/2012 08/2012 (15)	Δ	04/2012 09/2012 (16)	Δ	05/2012 10/2012 (17)	Δ	06/2012 11/2012 (18)	Δ
0	11.97%	26.61%	14.64%	22.40%	10.43%	22.22%	10.25%	30.65%	18.68%	19.53%	7.56%	15.63%	3.66%
1	11.39%	28.23%	16.84%	18.40%	7.01%	11.11%	-0.28%	3.23%	-8.16%	3.91%	-7.48%	3.91%	-7.48%
2	10.82%	16.94%	6.11%	13.60%	2.78%	5.56%	-5.27%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%
3	10.43%	1.61%	-8.82%	1.60%	-8.83%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%
4	10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%
5	9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%
6	9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%
7	9.04%	0.00%	-9.04%	0.00%	-9.04%	1.59%	-7.45%	1.61%	-7.42%	1.56%	-7.47%	1.56%	-7.47%
8	8.76%	8.06%	-0.69%	22.40%	13.64%	38.10%	29.34%	39.52%	30.76%	41.41%	32.65%	44.53%	35.77%
9	8.50%	18.55%	10.05%	21.60%	13.10%	21.43%	12.93%	25.00%	16.50%	33.59%	25.09%	34.38%	25.88%

Digit	Benford Rate	07/2012 12/2012 (19)	Δ	08/2012 01/2013 (20)	Δ	09/2012 02/2013 (21)	Δ	10/2012 03/2013 (22)	Δ	11/2012 04/2013 (23)	Δ	12/2012 05/2013 (24)	Δ
0	11.97%	24.22%	12.25%	39.06%	27.09%	51.59%	39.62%	42.86%	30.89%	38.71%	26.74%	36.59%	24.62%
1	11.39%	10.16%	-1.23%	10.94%	-0.45%	12.70%	1.31%	9.52%	-1.87%	8.87%	-2.52%	8.94%	-2.45%
2	10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%
3	10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%
4	10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%
5	9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%
6	9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%
7	9.04%	1.56%	-7.47%	1.56%	-7.47%	0.00%	-9.04%	0.00%	-9.04%	0.00%	-9.04%	0.81%	-8.22%
8	8.76%	36.72%	27.96%	22.66%	13.90%	7.14%	-1.61%	11.11%	2.35%	25.00%	16.24%	38.21%	29.45%
9	8.50%	27.34%	18.84%	25.78%	17.28%	28.57%	20.07%	36.51%	28.01%	27.42%	18.92%	15.45%	6.95%

Digit	Benford Rate	01/2013 06/2013 (25)	Δ	02/2013 07/2013 (26)	Δ	03/2013 08/2013 (27)	Δ	04/2013 09/2013 (28)	Δ
0	11.97%	27.42%	15.45%	12.00%	0.03%	0.00%	11.97%	0.00%	-11.97%
1	11.39%	2.42%	-8.97%	1.60%	-9.79%	0.00%	-11.39%	0.00%	-11.39%
2	10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%
3	10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%
4	10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%
5	9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%
6	9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%	0.00%	-9.34%
7	9.04%	8.87%	-0.16%	24.80%	15.77%	30.95%	21.92%	46.46%	37.42%
8	8.76%	42.74%	33.98%	44.80%	36.04%	54.76%	46.00%	50.39%	41.64%
9	8.50%	18.55%	10.05%	16.80%	8.30%	14.29%	5.79%	3.15%	-5.35%

Digit	Benford Rate	05/2013 10/2013 (29)	Δ	06/2013 11/2013 (30)	Δ	07/2013 12/2013 (31)	Δ
0	11.97%	0.00%	-11.97%	0.00%	-11.97%	0.00%	-11.97%
1	11.39%	0.00%	-11.39%	0.00%	-11.39%	0.00%	-11.39%
2	10.82%	0.00%	-10.82%	0.00%	-10.82%	0.00%	-10.82%
3	10.43%	0.00%	-10.43%	0.00%	-10.43%	0.00%	-10.43%
4	10.03%	0.00%	-10.03%	0.00%	-10.03%	0.00%	-10.03%
5	9.67%	0.00%	-9.67%	0.00%	-9.67%	0.00%	-9.67%
6	9.34%	0.00%	-9.34%	0.00%	-9.34%	1.55%	-7.79%
7	9.04%	51.16%	42.13%	65.89%	56.86%	72.09%	63.06%
8	8.76%	45.74%	36.98%	31.01%	22.25%	26.36%	17.60%
9	8.50%	3.10%	-5.40%	3.10%	-5.40%	0.00%	-8.50%

Table 4 - *Bendord's Law and LME Aluminium Third Digit Frequencies over rolling 6-Month Periods from 2011 to 2013*

Digit	Benford Rate	01/2011 06/2011 (32)	Δ	02/2011 07/2011 (33)	Δ	03/2011 08/2011 (34)	Δ	04/2011 09/2011 (35)	Δ	05/2011 10/2011 (36)	Δ	06/2011 11/2011 (37)	Δ
0	11.97%	11.38%	-0.59%	12.10%	0.13%	14.29%	2.32%	14.40%	2.43%	14.84%	2.88%	15.38%	3.42%
1	11.39%	7.32%	-4.07%	9.68%	-1.71%	8.73%	-2.66%	10.40%	-0.99%	9.38%	-2.01%	10.00%	-1.39%
2	10.82%	8.13%	-2.69%	9.68%	-1.14%	10.32%	-0.50%	12.00%	1.18%	12.50%	1.68%	11.54%	0.72%
3	10.43%	11.38%	0.95%	10.48%	0.05%	8.73%	-1.70%	8.00%	-2.43%	7.03%	-3.40%	7.69%	-2.74%
4	10.03%	9.76%	-0.27%	10.48%	0.45%	12.70%	2.67%	13.60%	3.57%	12.50%	2.47%	9.23%	-0.80%
5	9.67%	6.50%	-3.16%	7.26%	-2.41%	7.14%	-2.53%	8.00%	-1.67%	7.81%	-1.86%	6.92%	-2.74%
6	9.34%	9.76%	0.42%	8.06%	-1.27%	9.52%	0.19%	10.40%	1.06%	8.59%	-0.74%	7.69%	-1.64%
7	9.04%	13.82%	4.79%	12.90%	3.87%	12.70%	3.66%	10.40%	1.37%	12.50%	3.47%	13.08%	4.04%
8	8.76%	9.76%	1.00%	10.48%	1.73%	7.94%	-0.82%	6.40%	-2.36%	7.81%	-0.94%	9.23%	0.47%
9	8.50%	12.20%	3.70%	8.87%	0.37%	7.94%	-0.56%	6.40%	-2.10%	7.03%	-1.47%	9.23%	0.73%

Digit	Benford Rate	07/2011 12/2011 (38)	Δ	08/2011 01/2012 (39)	Δ	09/2011 02/2012 (40)	Δ	10/2011 03/2012 (41)	Δ	11/2011 04/2012 (42)	Δ	12/2011 05/2012 (43)	Δ
0	11.97%	16.41%	4.44%	17.19%	5.22%	16.54%	4.57%	15.27%	3.30%	14.40%	2.43%	12.80%	0.83%
1	11.39%	8.59%	-2.80%	7.81%	-3.58%	11.02%	-0.37%	9.45%	-1.93%	10.40%	-0.99%	8.00%	-3.39%
2	10.82%	10.94%	0.12%	10.94%	0.12%	9.45%	-1.37%	9.45%	-1.37%	8.00%	-2.82%	8.00%	-2.82%
3	10.43%	7.03%	-3.40%	9.38%	-1.06%	8.66%	-1.77%	9.82%	-0.61%	12.00%	1.57%	11.20%	0.77%
4	10.03%	10.16%	0.13%	7.81%	-2.22%	5.51%	-4.52%	8.36%	-1.67%	6.40%	-3.63%	7.20%	-2.83%
5	9.67%	7.81%	-1.86%	7.81%	-1.86%	9.45%	-0.22%	8.00%	-1.67%	10.40%	0.73%	10.40%	0.73%
6	9.34%	6.25%	-3.09%	6.25%	-3.09%	6.30%	-3.04%	7.64%	-1.70%	5.60%	-3.74%	8.00%	-1.34%
7	9.04%	14.84%	5.81%	14.06%	5.03%	14.96%	5.93%	13.82%	4.78%	13.60%	4.57%	16.00%	6.97%
8	8.76%	9.38%	0.62%	8.59%	-0.16%	8.66%	-0.10%	8.36%	-0.39%	8.00%	-0.76%	8.80%	0.04%
9	8.50%	8.59%	0.09%	10.16%	1.66%	9.45%	0.95%	9.82%	1.32%	11.20%	2.70%	9.60%	1.10%

Digit	Benford Rate	01/2012 06/2012 (44)	Δ	02/2012 07/2012 (45)	Δ	03/2012 08/2012 (46)	Δ	04/2012 09/2012 (47)	Δ	05/2012 10/2012 (48)	Δ	06/2012 11/2012 (49)	Δ
0	11.97%	8.87%	-3.10%	8.00%	-3.97%	6.35%	-5.62%	4.03%	-7.94%	6.25%	-5.72%	7.03%	-4.94%
1	11.39%	11.29%	-0.10%	11.20%	-0.19%	9.52%	-1.87%	8.87%	-2.52%	7.81%	-3.58%	7.81%	-3.58%
2	10.82%	9.68%	-1.14%	8.00%	-2.82%	10.32%	-0.50%	12.10%	1.27%	10.16%	-0.67%	13.28%	2.46%
3	10.43%	14.52%	4.08%	14.40%	3.97%	12.70%	2.27%	9.68%	-0.76%	9.38%	-1.06%	10.94%	0.50%
4	10.03%	8.87%	-1.16%	9.60%	-0.43%	11.11%	1.08%	12.10%	2.07%	10.16%	0.13%	10.16%	0.13%
5	9.67%	8.87%	-0.80%	7.20%	-2.47%	5.56%	-4.11%	7.26%	-2.41%	4.69%	-4.98%	4.69%	-4.98%
6	9.34%	6.45%	-2.89%	11.20%	1.86%	12.70%	3.36%	15.32%	5.99%	14.84%	5.51%	13.28%	3.94%
7	9.04%	13.71%	4.67%	13.60%	4.57%	12.70%	3.66%	12.10%	3.06%	14.84%	5.81%	12.50%	3.47%
8	8.76%	8.87%	0.11%	8.00%	-0.76%	8.73%	-0.03%	9.68%	0.92%	11.72%	2.96%	10.94%	2.18%
9	8.50%	8.87%	0.37%	8.00%	0.30%	10.32%	1.82%	8.87%	0.37%	10.16%	1.66%	9.38%	0.87%

Digit	Benford Rate	07/2012 12/2012 (50)	Δ	08/2012 01/2013 (51)	Δ	09/2012 02/2013 (52)	Δ	10/2012 03/2013 (53)	Δ	11/2012 04/2013 (54)	Δ	12/2012 05/2013 (55)	Δ
0	11.97%	9.38%	-2.59%	14.06%	2.09%	13.49%	1.52%	16.67%	4.70%	15.32%	3.35%	16.26%	4.29%
1	11.39%	4.69%	-6.70%	3.13%	-8.26%	2.38%	-9.01%	3.97%	-7.42%	3.23%	-8.16%	4.07%	-7.32%
2	10.82%	10.94%	0.12%	13.28%	2.46%	11.11%	0.29%	11.11%	0.29%	11.29%	0.47%	11.38%	0.56%
3	10.43%	12.50%	2.07%	10.16%	-0.28%	9.52%	-0.91%	10.32%	-0.12%	8.87%	-1.56%	8.94%	-1.49%
4	10.03%	9.38%	-0.66%	10.16%	0.13%	9.52%	-0.51%	11.11%	1.08%	14.52%	4.49%	15.45%	5.42%
5	9.67%	4.69%	-4.98%	4.69%	-4.98%	7.14%	-2.53%	4.76%	-4.91%	7.26%	-2.41%	7.32%	-2.35%
6	9.34%	14.06%	4.73%	10.16%	0.82%	10.32%	0.98%	7.94%	-1.40%	9.68%	0.34%	9.76%	0.42%
7	9.04%	12.50%	3.47%	13.28%	4.25%	15.87%	6.84%	14.29%	5.25%	13.71%	4.67%	13.01%	3.97%
8	8.76%	10.94%	2.18%	9.38%	0.62%	10.32%	1.56%	8.73%	-0.03%	6.45%	-2.31%	5.69%	-3.07%
9	8.50%	10.94%	2.44%	11.72%	3.22%	10.32%	1.82%	11.11%	2.61%	9.68%	1.18%	8.13%	-0.37%

Digit	Benford Rate	01/2013 06/2013 (56)	Δ	02/2013 07/2013 (57)	Δ	03/2013 08/2013 (58)	Δ	04/2013 09/2013 (59)	Δ	05/2013 10/2013 (60)	Δ	06/2013 11/2013 (61)	Δ	07/2013 12/2013 (62)	Δ
0	11.97%	14.52%	2.55%	11.20%	-0.77%	10.32%	-1.65%	7.87%	-4.09%	10.85%	-1.12%	8.53%	-3.44%	8.53%	-3.44%
1	11.39%	6.45%	-4.94%	6.40%	-4.99%	5.56%	-5.83%	3.94%	-7.45%	5.43%	-5.96%	5.43%	-5.96%	4.65%	-6.74%
2	10.82%	12.10%	1.27%	9.60%	-1.22%	10.32%	-0.50%	7.87%	-2.95%	10.08%	-0.74%	6.98%	-3.85%	6.98%	-3.85%
3	10.43%	7.26%	-3.17%	8.80%	-1.63%	12.70%	2.27%	12.60%	2.17%	14.73%	4.30%	17.83%	7.40%	16.28%	5.85%
4	10.03%	16.94%	6.90%	16.80%	6.77%	17.46%	7.43%	18.11%	8.08%	14.73%	4.70%	15.50%	5.47%	14.73%	4.70%
5	9.67%	7.26%	-2.41%	8.80%	-0.87%	9.52%	-0.14%	14.17%	4.51%	10.85%	1.18%	10.85%	1.18%	13.18%	3.51%
6	9.34%	8.87%	-0.47%	11.20%	1.86%	7.94%	-1.40%	9.45%	0.11%	6.98%	-2.36%	8.53%	-0.81%	10.85%	1.52%
7	9.04%	11.29%	2.26%	11.20%	2.17%	10.32%	1.28%	11.81%	2.78%	9.30%	0.27%	9.30%	0.27%	10.08%	1.04%
8	8.76%	7.26%	-1.50%	8.00%	-0.76%	7.14%	-1.61%	8.66%	-0.10%	10.85%	2.10%	10.85%	2.10%	8.53%	-0.23%
9	8.50%	8.06%	-0.44%	8.00%	-0.50%	8.73%	0.23%	5.51%	-2.99%	6.20%	-2.30%	6.20%	-2.30%	6.20%	-2.30%

In Table 4, Benfor's Law second digit rates, rather than third digit ones, are reported since, for the sake of simplicity of computations, we deleted first digits from the entire dataset when we dealt with the empirical third digit distribution. Thus, we can treat third digits as they were second digits. Of course, this expedient does not alter the value of Benford's Law analysis. Furthermore, this allow us to better and equally compare the two LME Aluminium sets of rates, reported in Table 3 and Table 4, respect to Benford's Law one.

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